Exam Seat No:_______________ Enrollment No:__________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 5SCO2MTC4 **Summer Examination-2014** **Date: 16/06/2014**

Subject Name:-Real Analysis-I

Branch/Semester:- M.Sc(Mathematics)/II **Time:02:00 To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places
(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) Suppose A_1 and A_2 be any algebras on X. Is $A_1 \cup A_2$ always an algebra on ? If no give an example. (02)

b) Show that countably additive measure m is monotonic. (02)

c) Let $X = \{1, 2, 3\}$, Is $A = \{\phi, X, \{1\}, \{2, 3\}\}$ an algebra on X ? (01)

d) Define σ −algebra. (01)

e) Define measurable function. (01)

Q-2 a) For any element $x \in R$, define $m: P(R) \to [0, \infty]$ by (05)

$$m(E) = \begin{cases} 1, & x \in E \\ 0, & x \notin E \end{cases},$$

Then show that m is a countably additive measure.

b) Show that the set of all measurable sets $\mathfrak{M}$ is an algebra in R. (05)

c) Let $\{E_i\}$ be an increasing sequence of measurable sets. Then prove that $m(\bigcup_n E_n) = \lim_n m(E_n)$. (04)

OR

Q-2 a) Let $\mathcal{A}$ be an algebra in $X(\neq \phi)$. Suppose $\{A_i\}_{i\geq i} \in \mathcal{A}$. Then prove that there exists a sequence $\{B_i\}_{i\geq i} \in \mathcal{A}$ such that (05)

(i) $\bigcup_i A_i = \bigcup_i B_i$

(ii) $B_j \cap B_k = \phi, j \neq k$.

b) Prove that countably additive measure m is countably sub-additive. (05)

c) Let E be a measurable subset of R. Then show that for every $\epsilon > 0$ arbitrary small there is an open set $O \supset E$ such that $m^*(O - E) < \epsilon$. (04)

Q-3 a) State and prove Littlewood's 3rd principle. (07)

b) Suppose $\{f_n\}_{n\geq 1}$ is a sequence of measurable functions on a measurable domain. Then prove that $\sup_n f_n$ & $\inf_n f_n$ are measurable. (04)

c) If $m^*(E) = 0$, then show that E is measurable. Is the converse true? (03)

OR

Q-3 a) Prove that the outer measure of an interval is its length. (07)

b) Let f be measurable and $f = g$ a.e. Then prove that g is measurable. (04)

c) Define measurable set. Show that ϕ and R are measurable. (03)

SECTION-II

Q-4 a) Suppose A and B are disjoint measurable sets with finite measures and f be bounded measurable function then show that (02)

$$\int_{A\cup B} f = \int_A f + \int_B f.$$

b) Let f be bounded in $[a,\ b]$. If f is Riemann integrable then prove that f is measurable and $R\int_a^b f = \int_{[a,\ b]} f$. (02)

c) Suppose ϕ is a measurable simple function with $\phi = 0$ a.e. Then show that $\int \phi = 0$. (02)

d) State Beppo-Levi's theorem. (01)

Q-5 a) Suppose ϕ and ψ are two measurable simple functions then prove that (05)

$$\int a\phi + b\psi = a\int \phi + b\int \psi.$$

b) Let f and g be two bounded measurable functions defined on a measurable set E of finite measure. If $f \geq g$ a. e., then prove that (05)

$$\int_E f \geq \int_E g.$$

c) Evaluate $\lim_{n\to\infty} \int_2^5 \frac{nx}{1+nx} dx$, using monotone convergence theorem. (04)

OR

Q-5 a) Let $\{f_n\}$ be a sequence of non-negative measurable functions defined on a measurable set E. Suppose $f_n(x) \to f(x), x \in E$ (pointwise). Then prove that $\int_E f \leq \underline{\lim} \int_E f_n$. (05)

b) Let f be a non-negative measurable function which is integrable on a measurable set E. Then prove that for each $\epsilon > 0$, arbitrary small there exists $\delta > 0$ such that for every measurable subset A of E with $m(A) < \delta$, we have $\int_A f < \epsilon$. (05)

c) Show that monotonically increasing functions are of bounded variation. (04)

Q-6 a) State and prove bounded convergence theorem. (07)

b) State and prove Lebesgue's dominated convergence theorem. (07)

OR

Q-6 a) Prove that a function $f \in BV[a,\ b]$ iff f can be expressed as the difference of two monotonically increasing functions on $[a,\ b]$. (07)

b) Let f be bounded measurable function on $[a,\ b]$. Set (07)

$$F(x) = \int_a^x f(t)\, dt + F(a)$$

then prove that $F'(x) = f(x)$ a. e. on $[a,\ b]$.

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